A Cone-Beam Filtered Backprojection Reconstruction Algorithm for Cardiac Single Photon Emission Computed Tomography

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Abstract—A filtered backprojection reconstruction algorithm was developed for cardiac single photon emission computed tomography with cone-beam geometry. The algorithm reconstructs cone-beam projections collected from "short scan" acquisitions of a detector traversing a noncircular planar orbit. Since the algorithm does not correct for photon attenuation, the algorithm is designed to reconstruct data collected over an angular range of slightly more than 180° with the assumption that the range of angles is oriented so as not to acquire the highly attenuated posterior projections of emissions from cardiac radiopharmaceuticals. This sampling scheme is performed to minimize the attenuation artifacts that result from reconstructing posterior projections. From computer simulations, we find that reconstruction of attenuated projections has a greater effect upon quantitation and image quality than any potential cone-beam reconstruction artifacts resulting from insufficient sampling of cone-beam projections. With nonattenuated projection data, cone-beam reconstruction errors in the heart are shown to be small (errors of at most 2%) which is the result of the heart being positioned near the central plane of the cone-beam geometry. Artifacts increase in the surrounding tissue at distances further from the central plane.

I. INTRODUCTION

Today, the rotating scintillation camera is the most widely used single photon emission computed tomography (SPECT) system in clinical nuclear medicine, and cardiac SPECT accounts for over 50% of all SPECT procedures. The improvement in SPECT sensitivity depends in part upon developing converging collimators, such as a cone-beam collimator, which can be used with large field of view detectors. Clinically, this technology has the potential to improve cardiac SPECT imaging, with improved lesion detectability and better diagnosis of ischemic heart disease.

In applying cone-beam tomography to cardiac imaging [1],[2], a cone-beam collimator is mounted to a camera so that the face of the camera remains parallel to the axis of rotation. The goal is to magnify the heart as much as possible over a large crystal area in order to maximize the sensitivity, while at the same time keeping the heart in the field of view through all rotation angles. The camera rotates in a noncircular orbit, with the center of the camera always pointing to the center of rotation, and closely follows the patient's body contour in order to optimize the projected spatial resolution.

Since accurate attenuation correction methods are impractical with present day technology, the approach has been to determine an optimum angular sampling range that would give minimum distortion if projection data are reconstructed using filtered backprojection algorithms without attenuation correction [3]–[6]. For cone-beam geometry, projection data are acquired over an angular range (180° plus the cone angle plus an additional small angle) that is sufficient to accurately reconstruct fan-beam projections of the central transaxial slice of the cone-beam geometry. However, the sampling is chosen so not to acquire the highly attenuated posterior projections of radiopharmaceutical emissions from the heart. The acquisition over this angular range is termed "short scan" acquisition because of less than the optimum 360° of data required for fan and cone-beam reconstruction algorithms.

Previously, cone-beam reconstruction algorithms [7]–[34] have been developed for various medical applications. In practical applications, cone-beam algorithms like the Feldkamp algorithm [9] attempt to obtain the best possible reconstruction with insufficient sampling of cone-beam projections. For sufficient sampling, the scanning trajectory must have at least one point of intersection for any plane passing through the reconstructed region of interest [20]; however, present clinical SPECT imaging systems scan so the focal point traces out a single planar orbit and thus, do not meet this requirement. Our application has the additional restriction of not being able to scan posterior views due to increased attenuation in those projections. Thus, the reconstruction of cardiac SPECT images from cone-beam projection data is obtained from projections sampled over a smaller angular range than that, for instance, of cone-beam tomography of the brain [35],[36].

In this paper a cone-beam filtered backprojection reconstruction algorithm for cardiac SPECT is developed to reconstruct projections obtained from "short scan" acquisitions of noncircular detector orbits. The paper first derives the "full scan" fan-beam reconstruction algorithm for noncircular orbits which was previously presented by Weinstein [37] and again rederived in our earlier paper [1]. These results are modified to obtain a new "short scan" fan-beam reconstruction algorithm for noncircular orbits. The development differs from our previous paper [1] in the use of a cone independent weight function which more accurately weights the doubly sampled data in the reconstruction of "short scan" data acquisitions.

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This fan-beam formulation is then extended to a cone-beam reconstruction algorithm using the technique presented by Feldkamp [19]. Computer simulations are presented which demonstrate the accuracy of the cone-beam reconstruction algorithm for cardiac application.

II. THEORY

A. Fan-Beam “Full Scan” Convolution Algorithm For Noncircular Orbit

We first present in this section the fan-beam “full scan” convolution algorithm for noncircular orbit due to Weinstein [37] and rederived again in our earlier paper [1]. The results presented here will be used in the following sections to derive new formulations for the “short scan” fan beam and cone-beam reconstruction algorithms for noncircular orbits.

It is assumed that the fan-beam projection data are collected with an angular-dependent focal point to center variation \( d(\alpha) \) shown in Fig. 1. The function \( f(x, y) \) is used to denote the cross-section of an object to be reconstructed from its fan-beam projections. The object is zero outside of a circle of radius \( R \).

A parallel projection of \( f(x, y), p_p(\theta, t) \), is the collection of line integrals of \( f(x, y) \) along the paths given by

\[
t = -x \sin \theta + y \cos \theta,
\]

for the angle \( \theta \).

A polar coordinate version \( f(r, \phi) \) of the original function \( f(x, y) \) can be reconstructed from its parallel projections using the following integral equation

\[
f(r, \phi) = \int_{-\infty}^{2\pi} \int_{-\infty}^{\infty} p_p(\theta, t) h[r \sin(\phi - \theta) - t] dt d\theta
\]

where \( h(t) \) is given by

\[
h(t) = \int_{-\infty}^{\infty} (|\mu|/2)e^{2\pi i\mu t} d\mu,
\]

and \( h(t) \) satisfies \( h(at) = h(t)/a^2 \).

Consider the fan-beam geometry in Fig. 1 in which the focal point to the center-of-rotation \( d(\alpha) \) varies as a function of angle \( \alpha \) and the focal length \( D \) remains fixed. A fan-beam projection in this system is denoted by \( p_f(\alpha, \xi) \). It can be seen from Fig. 1 that the fan-beam and parallel projections are related as follows:

\[
p_p(\theta, t) = p_f(\alpha, \xi)
\]

where

\[
t = \xi d(\alpha) Z,
\]

\[
\theta = \alpha + \tan^{-1}(\xi/D),
\]

\[
Z = (\xi^2 + D^2)^{-1/2}.
\]

Using (5) and (6), the reconstruction formula given by (2) for the parallel projection case can be implemented in the fan-beam coordinates \((\alpha, \xi)\). The components of the Jacobian for this transformation are

\[
\frac{\partial t}{\partial \xi} = Z^2 D^2 d(\alpha),
\]

\[
\frac{\partial t}{\partial \alpha} = d_\alpha(\alpha)\xi Z,
\]

\[
\frac{\partial \theta}{\partial \xi} = Z^2 D,
\]

\[
\frac{\partial \theta}{\partial \alpha} = 1
\]

where \( d_\alpha(\alpha) \) is the partial derivative of \( d(\alpha) \) with respect to \( \alpha \). These four equations can be combined to determine the Jacobian

\[
|J(\alpha, \xi)| = |Dd(\alpha) - d_\alpha(\alpha)\xi|Z^3 D.
\]

Using the coordinate transformation in (5) and (6) and the expression for the Jacobian in (12), we arrive at

\[
f(r, \phi) = \int_{0}^{2\pi} \int_{-\infty}^{\infty} p_f(\alpha, \xi)|Dd(\alpha) - d_\alpha(\alpha)\xi|Z^3 D
\]

\[
\cdot h\{r \sin[\phi - \alpha - \tan^{-1}(\xi/D)]
\]

\[
- \xi Zd(\alpha)\} d\xi d\alpha.
\]

Keep in mind that the angle \( \phi \) is measured from the positive \( x \)-axis. The argument of the filter \( h \) in (13) can be written as

\[
r \sin[\phi - \alpha - \tan^{-1}(\xi/D)] - \xi Zd(\alpha) = UZ(\xi' - \xi)
\]

where

\[
\xi' = r D \sin(\phi - \alpha)/[r \cos(\phi - \alpha) + d(\alpha)],
\]
Using (14) and the fact $h(at) = h(t)/a^2$, we see that

$$h\{r \sin[\phi - \alpha - \tan^{-1}(\xi/D)] - \xi Z d(\alpha)\} = h(\xi' - \xi)/(U Z)^2.$$  \hspace{1cm} (17)

The following is obtained when (17) is substituted into (13)

$$f(r, \phi) = \int_{0}^{\pi} g(\alpha, \xi')/U^2 \, d\alpha$$  \hspace{1cm} (18)

where

$$g(\alpha, \xi') = \int_{-R}^{R} p_f(\alpha, \xi') D d(\alpha) - d_n(\alpha) \xi' D / (D^2 + \xi^2)^{1/2} \cdot h(\xi' - \xi) \, d\xi,$$  \hspace{1cm} (19)

and $R$ is the value of $\xi$ for which $p_f(\alpha, \xi) = 0$ for all $|\xi| > R$. Equation (18) which was also previously derived in [1] is a filtered backprojection algorithm for fan-beam projections collected with an angular-dependent focal point to center variation.

**B. Fan-Beam “Short Scan” Reconstruction Algorithm For Noncircular Orbit**

The application of a filtered backprojection algorithm, like the one derived in the previous section, to “short scan” fan-beam data sets requires the premultiplication of the projections by a weight function that properly weights doubly sampled data. Our approach in [1] applied a weight function derived for “short scan” circular orbit acquisitions to the reconstruction of “short scan” projection data sets acquired from noncircular detector orbits. In simulations we found this gave significant reconstruction artifacts. We have investigated various approaches that could be used in developing a more accurate “short scan” algorithm for noncircular orbit acquisitions [41]. Here one method is presented which we have found to be accurate and computationally efficient.

Suppose the projection data is obtained over an angular range for $\alpha$ of $0 \leq \alpha \leq \alpha_{\text{max}}$ where $\pi \leq \alpha_{\text{max}} < 2\pi$. Over this angular sampling range there will be double sampling of some projection rays, that is, the line integral of the same ray is accomplished by placing the detector so that measurements are obtained at opposite ends of the projection ray. Here we emphasize that in our development we assume no attenuation.

From Fig. 2, we see that a projection ray with coordinates $(\alpha_1, \beta_1)$ has the same projection line integral as the projection ray with coordinates $(\alpha_2, \beta_2)$ if

$$\alpha_1 + \beta_1 = \alpha_2 + \beta_2 - \pi,$$  \hspace{1cm} (20)

$$d(\alpha_1) \sin[\beta_1] = -d(\alpha_2) \sin[\beta_2]$$  \hspace{1cm} (21)

where $\beta_1 = \tan^{-1}[\xi_1/D]$ and $\beta_2 = \tan^{-1}[\xi_2/D]$. Using (20) and (21) and knowing the range $\alpha_{\text{max}}$ of angles sampled, the doubly scanned regions in the projection sinogram can be determined.

First, let us consider the sinogram for fan-beam projections with circular detector orbit where the focal point to center distance $d(\alpha)$ is a constant. The sinogram is shown in Fig. 3 as a distribution of sampled coordinates $(\alpha, \beta)$ where $\alpha$ is the fan-beam projection angle and $\beta = \tan^{-1}[\xi/D]$ is the projection ray angle. If we sample $\beta_2$ for a constant projection angle $\alpha_2 = c$, using (20) and (21) a linear relationship between the coordinates $(\alpha_1, \beta_1 = -\beta_2)$ for the corresponding doubly sampled projection can be expressed as

$$\alpha_1 = -2\beta_1 + c - \pi.$$  \hspace{1cm} (22)

Using this relationship and assuming that $\alpha_{\text{max}} = \pi + 2\delta$ where $\delta$ is half the total fan angle, one can verify that the shaded regions in Fig. 3 are the coordinates of the doubly scanned projection data. The lower region is contained between the boundaries $\alpha_1 = 0$ and $\alpha_1 = -2\beta_1 + 2\delta$. The upper region is contained between the boundaries $\alpha_1 = -2\beta_1 + \pi$ and $\alpha_2 = \pi + 2\delta$. From (22) we see that the doubly sampled coordinates for a horizontal line in one region (i.e., $\alpha = \text{constant}$) is a straight line of slope $-2$ in the other region. This is illustrated in Fig. 3 for a doubly sampled pair of lines from which three pairs of double samples are identified as *, o, and x.

If a reconstruction is accomplished using the standard fan-beam reconstruction algorithm, and assuming projection data for $\pi + 2\delta < \alpha \leq 2\pi$ to be zero, reconstruction artifacts will result due to the double coverage of some projection data in the sampled region $(0 \leq \alpha \leq \pi + 2\delta)$. One might think that the reconstruction can be improved by setting the data to zero in a region $(\alpha_2 + 2\beta_2 > \pi)$ which is doubly scanned. This was shown by Naparstek [38] to produce severe streaks in the reconstructed image. The streak artifacts are due to the sharp cutoff in some projections yielding high frequency components that are enhanced by the reconstruction convolution function. Parker [39] reduced the sharp cutoff by applying a smoothing window function $W_{\alpha}(\beta)$ to the sinogram image so that

$$W_{\alpha_1}(\beta_1) + W_{\alpha_2}(\beta_2) = 1$$  \hspace{1cm} (23)
where \((\alpha_1, \beta_1)\) and \((\alpha_2, \beta_2)\) satisfy (20) and (21) with \(d(\alpha)\) constant. This approach significantly improved the image quality. Another approach [40] is to set any doubly sampled data bins to zero after the application of the reconstruction convolver. This also improves image quality but is arrived at heuristically and is not mathematically exact.

Several different window functions can easily be derived for the approach proposed by Parker [39] if an analytical relationship can be expressed between the coordinates of the doubly sampled data. For circular orbits this is easily done as we showed in arriving at the expression in (22). For noncircular orbits, one can immediately see from (20) and (21) that analytical expressions are not readily available since any relationship must include arc sine terms. Therefore, a window function as defined in (23) is hard to implement in general for noncircular orbits since the coordinates of the doubly sampled projections can only be determined numerically. Also, notice that from (20) and (21), the minimum required scanning range of angles for circular orbit (i.e., the minimum range of angles that samples all projection rays at least once) is \(180^\circ + 2\delta\). Whereas, for noncircular orbits the minimum scanning range cannot in general be determined analytically from (20) and (21).

Instead of determining numerically the doubly sampled region, our approach has been to determine an analytical solution to the "short scan" problem for noncircular detector orbits [41]. If we go back to the coordinate transformation from parallel to fan-beam geometry in (6), we can set \(\theta = \delta\) (= half the total fan angle) for the parallel projection angle \(\theta\). In Fig. 4 this gives the lower line: \(\alpha = -\beta + \delta\). For \(\theta = \delta + \pi\), we obtain the upper line: \(\alpha = -\beta + \delta + \pi\). The fan-beam projection coordinates between these two lines represent a complete data set without any duplicated samples. For the regions labeled 1 in Fig. 4, the lower region lies between these two lines and the corresponding doubly sampled region (the upper region) lies outside, and likewise for the doubly sampled regions labeled 2.

From (5) and (6), one observes that \(t\) varies with the focal point to center-of-rotation distance \(d(\alpha)\), while \(\theta\) does not. If a weight function \(W(\theta)\) is designed so that it depends only on \(\theta\) and not on \(t\), then the fan-beam weight function \(W[\alpha + \tan^{-1}(\xi/D)]\) obtained through the coordinate transformation is orbit independent. For parallel geometry, the only requirement is that the weight function satisfies

\[
W(\theta) + W(\theta + \pi) = 1. \tag{24}
\]

The function can even be a zero-one function and it will not introduce any high-frequency components, because \(W(\theta)\) is constant with respect to the filtering implemented along the projection coordinate \(t\). However, after \(W(\theta)\) is transformed to the fan-beam coordinates: \(W[\alpha + \tan^{-1}(\xi/D)]\), a zero-one window function will introduce high frequency components. In the fan-beam reconstruction algorithm, the filtering (19) is performed with respect to \(\xi\) for each fixed \(\alpha\). Therefore, it is necessary that the weight function \(W[\alpha + \tan^{-1}(\xi/D)]\) be smooth at the boundaries of the doubly sampled regions, that is, the derivative \(dW[\alpha + \tan^{-1}(\xi/D)]/d\xi\) should be continuous so that \(W\) has a smooth transition at the boundary in the direction of the coordinate \(\xi\).

Analogous to Parker's approach [39], a smooth weight function is obtained which eliminates the sharp boundary transitions in the fan-beam variable \(\xi\). For a small transition band \(\gamma\), the weight function is defined as follows

\[
W(\theta) = \begin{cases} 
\sin^2\left(\frac{(\theta - \delta)}{\gamma}\right) & \text{if } \delta \leq \theta \leq \gamma + \delta, \\
1 & \text{if } \gamma + \delta \leq \theta \leq \pi + \delta, \\
\cos^2\left(\frac{(\theta - \delta - \pi)}{\gamma}\right) & \text{if } \pi + \delta \leq \theta \leq \pi + \gamma + \delta, \\
0 & \text{otherwise.}
\end{cases} \tag{25}
\]

We see from Fig. 5 that the parameter \(\gamma\) is an additional fan-beam sampling angle. It is necessary to scan over \(\pi + \gamma\)
2\delta + \gamma, instead of \pi + 2\delta required for circular detector orbits. The larger the angle \gamma is, the smoother the weight function \( W(\theta) \) will be. The selection of \gamma depends upon the data and the system bandwidth. Also, the data in the unshaded regions of Fig. 5 between 0 and \( \pi + 2\delta + \gamma \) are not used but are set to zero by the weight function in (25). The requirement to sample an additional range of projection angles \gamma and to throw away some sampled data is the price we pay to use an orbit independent, smooth weight function \( W(\theta) \) which offers an analytical solution to the “short scan” problem for noncircular orbits.

Using the weight function in (25), we modify (18) and (19) to obtain the following fan-beam “short scan” algorithm

\[
f(r, \phi) = \int_0^{\pi + 2\delta + \gamma} q(\alpha, \xi') U^2 d\alpha
\]

(26)

where

\[
q(\alpha, \xi) = \int_{-R}^{R} W[\alpha + \tan^{-1}(\xi/D)] p_f(\alpha, \xi) \\
\cdot \left| Dd(\alpha) - d_{\xi}(\alpha) \xi(D/D^2 + \xi^2)^{1/2} \right| \\
\cdot h(\xi' - \xi) d\xi.
\]

(27)

The projection data point \( p_f(\alpha, \xi) \) is premultiplied by the weight function \( W \) defined in (25) for \( \theta = \alpha + \tan^{-1}(\xi/D) \). The result is then multiplied by the other multiplicative factors given in (27) which include the derivative \( d_{\xi}(\alpha) \). Then the preweighted projections are filtered and backprojected using (26). In practice, the derivative \( d_{\xi}(\alpha) \) is calculated using a finite difference approximation.

C. Cone-Beam “Short Scan” Reconstruction Algorithm
For Noncircular Orbit

The “short scan” fan-beam algorithm in (26) and (27) of the previous section is used to derive a cone-beam “short scan” reconstruction algorithm for noncircular orbits. The approach used is identical to the method presented by Feldkamp [19] and also used by us in our earlier derivation [1]. The results here differ from our earlier work in that the extension to cone-beam geometry is based upon a more accurate “short scan” fan-beam reconstruction algorithm for noncircular orbits. Keep in mind that our previous discussion concerning doubly scanned data only applies for the central slice in the cone-beam geometry. For cone-beam projections off the central plane, it is impossible to obtain doubly sampled projections for a single planar orbit acquisition (the orbit of the focal point of the cone-beam geometry remains in a plane) even if projections are sampled over 360°. Therefore, the discussion in the previous section concerning a “short scan” reconstruction algorithm is, in a strict sense, only valid for the central slice of the cone-beam geometry. However, our approach in obtaining a “short scan” algorithm for cone-beam geometry is to extend the fan-beam “short scan” algorithm to cone-beam geometry using the method proposed by Feldkamp [19] in his development of a “full scan” cone-beam reconstruction algorithm. The derivation assumes that planes off the central slice are approximately sampled by fan-beam projections. Using this approach we are assuming that an accurate “short scan” algorithm is a necessary approximation even for noncentral slices. Simulations verify the fact that the reconstruction results obtained by applying the Feldkamp algorithm to the “short scan” data are worse than those obtained using the “short scan” algorithm derived in this section.

To develop the algorithm, we use the cone-beam geometry shown in Fig. 6. The three-dimensional function \( f(x, y, z) \) to be reconstructed is represented in cylindrical coordinates as \( f(r, \phi, z) \). From (26), we know that the fan-beam reconstruction at the midplane for the cone-beam projections \( p_c(\alpha, \xi, \zeta) \) is given by

\[
f(r, \phi, z = 0) = \int_0^{\pi + 2\delta + \gamma} q(\alpha, \xi', \zeta = 0) U^2 d\alpha
\]

(28)

where \( \xi' \) and \( U \) are given by (15) and (16) and

\[
q(\alpha, \xi', \zeta = 0) = \int_{-R}^{R} W[\alpha + \tan^{-1}(\xi/D)] p_c(\alpha, \xi, \zeta = 0) \\
\cdot \left| Dd(\alpha) - d_{\xi}(\alpha) \xi(D/D^2 + \xi^2)^{1/2} \right| \\
\cdot h(\xi' - \xi) d\xi.
\]

(29)

Now consider a plane shown in Fig. 7(a) which is not the midplane and a rotation \( \delta \alpha' \) in Fig. 7(b) about an axis \( z' \) perpendicular to this plane. Then a reconstructed differential \( \delta f \) [19] is given by

\[
\delta f(r, \phi, z) = q(\alpha', \xi', \zeta') U^2 \delta \alpha'
\]

(30)

where

\[
\zeta' = zD/[r \cos(\phi - \alpha) + d(\alpha)].
\]

(31)
Fig. 6. Cone-beam geometry used for imaging the heart.

\[ U'' = U\left(D^2 + \zeta'^2\right)^{1/2}/D. \] (32)

\[ q'(\alpha', \xi', \zeta') = \int_{-R}^{R} \left[ p_{\alpha'}(\alpha', \xi, \zeta') \right] \cdot \left[ Dd'(\alpha') - d_{\alpha'}(\alpha')\xi D' / \left(D^2 + \xi'^2\right)^{1/2} \right] \cdot h(\xi' - \xi) d\xi \] (33)

Notice that we replaced \( D \) by \( D' \) in (29). Substituting \( D' = \left(D^2 + \zeta'^2\right)^{1/2} \), \( d'(\alpha') = \left(D^2 + \zeta'^2\right)^{1/2} d(\alpha)/D \), and \( d_{\alpha'}(\alpha') = d_{\alpha}(\alpha) \left(D^2 + \zeta'^2\right)^{1/2} \) into (33) gives

\[ q'(\alpha, \xi', \zeta') = \int_{-R}^{R} \left[ p_{\alpha}(\alpha, \xi, \zeta') \right] \cdot \left[ Dd(\alpha) - d_{\alpha}(\alpha)\xi / D^2 \right] \cdot \left(D^2 + \zeta'^2\right)^{3/2} / \left(D^2 + \zeta'^2 + \zeta^2\right)^{1/2} \cdot h(\xi' - \xi) d\xi. \] (34)

and using the fact \( d\alpha' = d\alpha D / \left(D^2 + \zeta'^2\right)^{1/2} \) gives

\[ q'(\alpha, \xi', \zeta') = \int_{-R}^{R} \left[ p_{\alpha}(\alpha, \xi, \zeta') \right] \cdot \left[Dd(\alpha) - d_{\alpha}(\alpha)\xi / D^2 \right] \cdot \left(D^2 + \zeta'^2\right)^{3/2} / \left(D^2 + \zeta'^2 + \zeta^2\right)^{1/2} \cdot h(\xi' - \xi) d\xi. \] (35)

A cone-beam reconstruction algorithm is derived by integrating over all projections sampled over \( \alpha \). Simplifying (34) and (35), we obtain

\[ f(r, \phi, z) = \int_{0}^{\pi/2} q'(\alpha, \xi', \zeta') / U^2 d\alpha \] (36)

The convolved projection in (37) at the angle \( \alpha \) is added to the value at the point \((r, \phi, z)\) as specified by (36). The point \((r, \phi, z)\) lies along the ray defined by the projection coordinates \( \xi' \) from (15) and \( \zeta' \) from (31).

The result in (37) differs in two ways from our earlier result [1]. First, the weight function \( W \) and the limits of integration are such that the central slice is reconstructed accurately for noncircular "short scan" acquisitions. Second, the term \( \tan^{-1} \left( \xi / \left(D^2 + \zeta'^2\right)^{1/2} \right) \) in the argument of the weight function \( W \) includes the transformation to cone-beam geometry which was previously ignored.
A. 5.7, and 4.2 cm and the outer intraventricular blood chamber emission source using a modification of the two-dimensional plane made an angle of \(28.6^\circ\) with a coronal slice passing through the center of the ellipsoid. The left ventricle and the plane had semiaxes of 2.7, 4.2, and 2.7 cm. A spherical defect of 0.6 cm. The outer left ventricular wall had semiaxes of 4.2, 0.6 cm. The left ventricle and the intraventricular blood chamber were both centered at (0, -4.8, 0.6 cm). The outer left ventricular wall had semiaxes of 4.2, 5.7, and 4.2 cm and the outer intraventricular blood chamber had semiaxes of 2.7, 4.2, and 2.7 cm. A spherical defect of radius equal to 0.72 cm was located in the ventricular wall with center at (3.36, -5.7, 1.2 cm).

Nonattenuated cone-beam projections were formed from the summation of line integrals over ellipsoids of constant emission source using a modification of the two-dimensional phantom generator, PHANL, in the RECLBL Library [42]. The attenuated projections were obtained by integrating the source intensity along appropriate lines such that at each source point the intensity is multiplied by an attenuation factor which is the exponential of the negative of the line integral of the ellipsoid attenuators between the source and the detector. All projections were formed as \(64 \times 64\) arrays with 6 mm square projection bins. For the cone-beam geometry, a focal length of 54 cm was assumed. Both circular and noncircular orbit data sets were formed. The circular orbit has a radius of 26.5 cm. The noncircular orbit was an ellipse with major and minor semiaxes of 26.5 and 23.5 cm, respectively. For the “full scan” simulations, 128 views were sampled over \(360^\circ\). For the “short scan” simulations, 83 views were acquired, which included 64 views over \(180^\circ\), plus 14 views sampled over an additional \(40^\circ\) equal to the cone angle, plus 5 more views required by the smoothing window in (25). There was no noise added to the projections.

Two simulation studies were performed. In the first study, projections of the source distribution were formed assuming no photon attenuation. Reconstructions of parallel projections (128 projections over \(360^\circ\)) were compared with reconstructions of “full scan” circular (128 projections), “short scan” circular (83 projections) and “short scan” noncircular (83 projections) cone-beam data sets.

In the second study, attenuation was included, and attenuated projections were formed using the attenuation distribution described above and illustrated in Fig. 8. For the parallel \(180^\circ\) reconstruction, 64 projections were formed starting at right lateral and traversing anteriorly to left lateral. For the circular and noncircular “short scans”, the projections were formed starting \(20^\circ\) right posterior oblique of right lateral such that the \(40^\circ\) of additional angles required for the “short scan” reconstruction algorithms were equally distributed between left posterior oblique and right posterior oblique. The parallel \(180^\circ\) reconstruction was compared to a cone-beam “full scan” circular, “short scan” circular, and “short scan” noncircular reconstructions. The “short scan” data were reconstructed using the algorithm in (36). The “full scan” data were reconstructed by modifying the algorithm to backproject over the full \(360^\circ\) with the window function set to one.

A 3 \(\times\) 3 convolution kernel with 1/3 in the center, 0 at each corner, and 1/6 everywhere else was first applied to all 64 \(\times 64\) projection arrays. Then the projections were reconstructed into a \(64 \times 64 \times 64\) cone array with a voxel width of 6 mm. The low-pass filter was applied to give a smooth transition between different source regions as would appear in nuclear medicine images where the source distributions are blurred by the system detector response. However, in practice the projections are not blurred by one simple blurring function as were done in the simulations, but the detector response is space varying such that image blurring increases with distance from the collimator.

B. Results

Fig. 9 shows the results of the first study. In simulating projections without attenuation, this study focuses on the differences between “short” and “full scan” reconstructions. The parallel reconstruction is used as a standard for evaluating...
The reconstruction artifacts associated with single planar orbit cone-beam tomography. From the profiles in Fig. 9, one sees that the reconstructions of the midplane (transverse slice 32) are equivalent, as one would expect, for all reconstructions. Remember that the reconstruction of the midplane for cone-beam geometry is equivalent to a fan-beam reconstruction. Therefore, "short scan" cone-beam reconstructions of the midplane should be equivalent to "full scan" cone-beam and parallel reconstructions if for the "short scan" acquisitions the central slice is sampled with a sufficient set of fan-beam projections for which the reconstruction algorithm in (36) gives an accurate reconstruction.

The cone-beam reconstructions of transaxial slices off the central slice are approximation due to the nature of the incomplete data sampling. However, one observes from the profiles in Fig. 9 that for a short distance off the midplane, such as transverse slice 35 which is 1.8 cm above the midplane, the cone-beam "full" and "short scan" reconstructions appear to be equivalent with the parallel reconstruction. Differences become more apparent for slices further from the midplane.

These differences are better seen in coronal slice 23 shown in Fig. 9 which is a distance of 9.5 voxels anterior of the central coronal slice. One immediately notices truncation artifacts in the cone-beam reconstructions which are at the inferior and superior boundary (arrow labeled A) of the cone-beam geometry (see [43] for a discussion of these artifacts). In addition, the profile for the parallel reconstruction has a higher amplitude than the profiles for the "full" and "short scan" cone-beam reconstructions. If we assume that the parallel reconstruction is accurate, one would infer that the differences in the profiles are due to the insufficient sampling of the cone-beam projections. Differences between "full" and "short scan" cone-beam reconstructions are more apparent in background tissue where low amplitude artifacts are seen extending horizontally from the inferior and superior walls of the heart.

Errors were measured quantitatively for both the background tissue and the heart region in coronal slice 23. Errors were measured by choosing a point in the background tissue located in the coronal slice at the level of the central transaxial plane (transaxial slice 32) and calculating the maximum absolute difference between the reconstruction at this point and at all points in the background tissue of the coronal slice off the central plane. By normalizing this to the point in the central plane which should be reconstructed accurately, maximum percent relative errors in the background tissue were calculated to be 1.4, 8.4, and 9.7% for "full scan," "short scan" circular, and "short scan" noncircular cone-beam reconstructions, respectively. Similar measurements were made for the heart region with the reference point chosen to be located in the heart and to lie within transaxial slice 32. Absolute differences were calculated between the reference point and all other points within the heart but not lying in the transaxial slice 32. The maximum percent relative error in all cases ("full scan," "short scan" circular, and "short scan" noncircular) resulted in much lower errors of at most 2%. It appears that the application of an approximate cone-beam reconstruction algorithm to "short scan" data sets, results in reconstruction artifacts for non central slice voxels that are not seen in "full scan" reconstructions. However, for the heart region these errors appear to be small.

The reconstructions of the attenuated projections and profiles are shown in Fig. 10. The profiles in both Figs. 9 and 10 are scaled to the same minimum and maximum value. From the profiles in Fig. 10 we see that intensity in the heart is nonuniform with a decrease in intensity by as much as a factor of three. Notice from the profiles shown in the first and last row that 360° reconstruction [cone (full)] has the lowest contrast between the heart and background tissue. This seems to collaborate with other studies of 180° versus 360° reconstructions [3]–[6] which recommend against "full scan" reconstructions. In addition, several other artifacts due to the effects of attenuation are seen. For example, differences in intensity between lungs and soft tissue are sufficient for one to see shadows of the lung even though the source is identically distributed in the two tissues. This is due to the large difference in the attenuation coefficients between lung (0.042 cm⁻¹) and soft tissue (0.125 cm⁻¹). Also, dark regions are seen in the background tissue extending laterally from each side of the heart. These artifacts differ in contrast between "full" and "short scan" reconstructions. It is helpful to convince oneself.
that all of these artifacts are caused by attenuation effects by comparing with the results in Fig. 9 where these artifacts are not shown.

Comparison of profiles for coronal slice 23 in Fig. 10 shows that the contrast is slightly reduced in the cone-beam "short scan" reconstructions compared to the parallel reconstructions. This finding seems to correlate with what was observed in Fig. 9 and what one would expect due to the inherent approximation in the cone-beam reconstruction algorithm. That expectation is that cone-beam reconstructed intensities for voxels that are off the central slice of the cone-beam geometry are not as accurate as parallel reconstructed intensities.

IV. DISCUSSION

A filtered backprojection reconstruction algorithm for cone-beam tomography of the heart was developed to reconstruct "short scan" data sets that are obtained using detectors rotating in planar noncircular orbits. For the heart, which can be positioned near the central plane of the cone-beam geometry, the reconstructions show very little geometric distortions. The attenuation of projection data has a greater effect upon quantization and image quality than any potential cone-beam reconstruction artifacts.

The reconstruction algorithms that are presently used clinically for cardiac SPECT do not correct for attenuation but instead select angles that minimize attenuation reconstruction artifacts for projections reconstructed using filtered backprojection algorithms. The reason is due to the difficulty of accurately correcting for attenuation which requires a measure of an attenuation map of the thorax combined with an iterative reconstruction algorithm. The iterative algorithms increase computation time and the measurement of the distribution of attenuation coefficients requires additional scanning time for most SPECT systems.

In developing cone-beam tomography of the heart, we took a similar approach by deriving a filtered backprojection reconstruction algorithm which does not correct for photon attenuation. In applying the algorithm to clinical data [2], we have found as have previous investigators [3]–[6] that the best results are obtained if one does not reconstruct the highly attenuated posterior projections of $^{201}\text{Tl}$ emissions from the heart. Therefore, the algorithm was designed to reconstruct "short scan" data sets with the specification that the central slice, which is sampled as a fan-beam projection data set, is reconstructed accurately. In order to use a filtered backprojection algorithm for noncircular orbits, a different method from that used with previous circular orbit algorithms [38], [39] had to be developed. A necessary feature in any "short scan" reconstruction algorithm is to be able to apply the reconstruction filter to the sinogram image across boundaries of doubly sampled regions in a way that it does not introduce high frequency artifacts. This is accomplished by applying a smooth weight function to the data at the boundaries such that the backprojection of filtered projections for projection bins corresponding to single and doubly sampled data have equal contribution to the reconstruction. The weight function which we presented in this paper for this purpose is orbit independent and thus can reconstruct projections acquired from any noncircular orbit.

The use of an orbit independent weight function imposes a penalty in that the "short scan" reconstruction algorithm requires an additional small range of angular samples and does not utilize a small fraction of the data. From the results of simulations, it is reasonable to expect that only five additional projection angles would be required. This amounts to approximately three minutes of additional scanning time. Also, one should keep in mind that the additional scanning range will more than likely give projections in which the source in the heart has greater attenuation. For a large patient, this attenuation will become more severe. Therefore, to minimize the attenuation artifacts it becomes important to optimally select the first projection angle, as it is also important for $180^\circ$ parallel projection acquisitions. The selection of the first projection angle can vary from patient to patient since the positioning in the chest of the left ventricle in relation to the torso varies in each patient.

In earlier work [1] we had used a weight function which was derived for circular orbits [39] to reconstruct "short scan" noncircular orbit projection data. We found in computer simulations that this gives significant reconstruction artifacts both in and off the central plane; whereas, the simulations...
presented in this paper show that the new orbit independent weight function gives accurate reconstruction of the central transaxial slice and minimum error in the cardiac region off the central plane. Also, its implementation is as efficient as the circular orbit weight function. We realize that if the reconstruction algorithm is accurate for the central slice, it is not guaranteed to be accurate off the central slice since the Feldkamp derivation [19] gives only an approximate cone-beam reconstruction algorithm. However, we have found through computer simulations that the extension of a more accurate formulation for the central slice does improve the overall accuracy of the cone-beam reconstruction algorithm. We do point out though that it is difficult in patient studies to distinguish qualitative differences in the reconstructions between those obtained using the new weight function and those obtained using the one derived for circular orbits because the noise and attenuation in nuclear medicine images masks the inaccuracies in the reconstruction algorithm.

In comparing the cone-beam reconstructions with parallel and fan-beam reconstructions, one can infer from the simulations that a point source on the central plane of the cone-beam geometry is reconstructed with a symmetric point response and this point response becomes more asymmetric as the source is axially removed further from the central plane [32]. It is known from simulations that the cone-beam reconstruction of some objects like the Defrise phantom (a series of equally spaced discs orthogonal to the axis of rotation) shows significant cross-talk between transaxial slices [43]. These simulations indicate that improved axial filtering is needed to obtain a more isotropic and spatially invariant point response. Another approach to improving cone-beam tomography is to use nonplanar detector orbits [44], but requires modification to present SPECT detector systems.

Both the cone-beam and parallel geometry reconstruction algorithms were implemented on a SUN SPARC-I (12 MIPS) computer in our laboratory. A 64 x 64 x 64 cone-beam reconstruction from 83 64 x 64 projections takes 24 min of CPU time as compared to 4 min to obtain a 64 x 64 x 64 parallel geometry reconstruction from 64 projection angles. From this we see that the cone-beam reconstruction time can be a factor of six times longer than the time to reconstruct parallel geometry projections. Continued advancement in computing hardware will improve cone-beam reconstruction time and will become clinically more acceptable.

Even though work continues to improve cardiac SPECT imaging through new attenuation correction techniques using simultaneous emission-transmission tomography [45] and iterative attenuation [27] and point response correction algorithms [46], it is felt that filtered backprojection algorithms, such as the one presented in this paper, applied to "short scan" data acquisitions will continue to have significant use in clinical nuclear medicine. At least in the near future, attenuation correction algorithms will not be implemented on clinical SPECT systems, and as such, techniques will be employed to minimize attenuation artifacts. It has been hoped that the replacement of $^{201}$Tl with $^{99m}$Tc labeled radiopharmaceuticals for SPECT imaging of the heart would increase photon statistics and improve attenuation reconstruction artifacts because of the higher energy photons of 140 keV for $^{99m}$Tc compared to 70 keV for $^{201}$Tl. It was also speculated that 360° reconstruction would yield optimum image quality. However, from simulations [47] and preliminary clinical results, it appears that reconstructions of posterior projections of emissions from $^{99m}$Tc radiopharmaceuticals significantly degrade image quality as it does for $^{201}$Tl, necessitating the use of "short scan" reconstruction algorithms.

The simulations presented in this paper show that a filtered backprojection reconstruction algorithm for cone-beam tomography of the heart can be developed which does not significantly degrade image quality when compared with parallel geometry reconstructions. In the application of this algorithm to patient and phantom data we showed [2] that cone-beam tomography can significantly improve SPECT imaging of the heart because of the increased sensitivity and resolution that can be obtained with using cone-beam collimators. Optimal image quality is obtained by integrating hardware and software in a way that optimizes reconstruction algorithms, collimator design [48], orbit specifications, and calibration techniques [49] so that the effects of photon attenuation, limited angular sampling, and data truncation are minimized. The research has shown that the technical problems of cone-beam tomography can be solved to obtain diagnostically useful images using present day off-the-shelf cone-beam collimators mounted to rotating gamma camera SPECT systems. Clinically, cone-beam tomography of the heart offers significant potential for improving lesion detection and diagnosis of ischemia in cardiac imaging.

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REFERENCES


